HOMEWORK #6 SELECTED SOL'NS

Section 1.8 #7b

Let $a, b, c \in \mathbb{N}$ be such that $\gcd(a,b) = 1$.
If $a | bc$ then $a | c$.

Proof: Let $a, b, c \in \mathbb{N}$ be such that $\gcd(a,b) = 1$ and $a | bc$. Since $\gcd(a,b) = 1$ $\exists\, k, \ell \in \mathbb{Z}$ such that

$$ka + \ell b = 1 \implies kac + \ell bc = c.$$

Since $a | bc$ $\exists\, m \in \mathbb{Z}$ such that $ma = bc$.

Thus

$$kac + \ell bc = c \implies kac + \ell ma = c$$
$$\implies a(kc + \ell m) = c$$

So $a | c$, as desired. $\blacksquare$

Section 3.2 #6c

Let V be the relation on $\mathbb{R}$ given by

$$xVy \Leftrightarrow (x=y \text{ or } xy=1).$$

Show V is an equivalence relation.

Proof: By definition xVx $\forall x \in \mathbb{R}$, so V is reflexive.

Let $x,y \in \mathbb{R}$ with xVy. Then either $x=y$ or $xy=1$. If $x=y$ then yVx and if $xy=1$ then $yx=1$ so yVx. In any case yVx so V is symmetric.

Suppose $x,y,z \in \mathbb{R}$ are such that

$$xVy \text{ and } yVz.$$

Since xVy there are two cases:

<u>Case 1</u>: if $x=y$ then

$$\begin{aligned} yVz &\Rightarrow (y=z \text{ or } yz=1) \\ &\Rightarrow (x=z \text{ or } xz=1) \\ &\Rightarrow xVz. \end{aligned}$$

<u>Case 2</u>: if $xy=1$ then (note $y \neq 0$ since $xy=1$)

$$\begin{aligned} yVz &\Rightarrow (y=z \text{ or } yz=1) \\ &\Rightarrow (xz=1 \text{ or } yz=xy) \\ &\Rightarrow (xz=1 \text{ or } z=x) \\ &\Rightarrow xVz. \end{aligned}$$

In either case xVz. Thus, V is transitive.

We conclude V is an equivalence relation. $\boxtimes$

(more 6c)

Give the equiv class of $3, \frac{-2}{3}, 0$.

$\overline{3} = \{3, 1/3\}$

$\overline{-2/3} = \{-2/3, -3/2\}$

$\overline{0} = \{0\}$.